Dephrasure channel and superadditivity of coherent information

Felix Leditzky

(JILA & CTQM, University of Colorado Boulder)

joint work with Debbie Leung and Graeme Smith

arXiv:1806.08327

Beyond IID, Cambridge, UK

26 July 2018

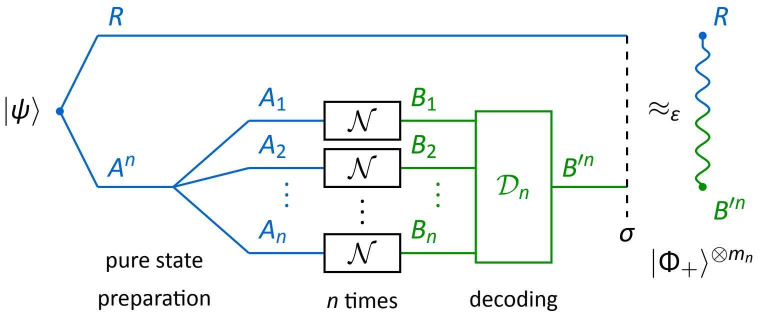
Outline

Table of Contents

Entanglement generation

- Entanglement can be used as a resource in:
  teleportation, dense coding, entanglement-assisted communication, ...
- Assume Alice and Bob can communicate via a **noisy quantum channel** $\mathcal{N} : A \to B$.
- **Entanglement generation:** Use the noisy channel and local operations to generate entanglement between the parties.
- We can allow for **one-way classical communication** without changing the task.

Entanglement generation



- **Goal:** Generate m_n ebits $|\Phi_+\rangle \sim |00\rangle + |11\rangle$ through n uses of the quantum channel $\mathcal{N}$.
- **Alice** prepares $|\psi\rangle_{RA^n}$ and sends A^n to **Bob** through $\mathcal{N}^{\otimes n}$.
- **Quantum capacity** $Q(\mathcal{N}) := \sup \left\{ \lim \frac{m_n}{n} \text{ s.t. } \varepsilon \xrightarrow{n\to\infty} 0 \right\}$.

Quantum capacity

- **Coding theorem**: [Lloyd 1997; Shor 2002; Devetak 2005]

$$Q(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} Q^{(1)}(\mathcal{N}^{\otimes n}) \qquad (*)$$

  where the **channel coherent information** $Q^{(1)}(\cdot)$ is defined as

$$Q^{(1)}(\mathcal{N}) := \max_{|\psi\rangle_{A'A}} I(A'\rangle B)_{(\mathrm{id}\otimes\mathcal{N})(\psi)}.$$

  with the coherent information $I(P\rangle Q)_\rho = S(Q)_\rho - S(PQ)_\rho$.

- **Regularized formula** $(*)$ in general **intractable to compute**.

- Notorious example: Qubit depolarizing channel

$$\mathcal{D}_p(\rho) := (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z).$$

- **Known:** $Q(\mathcal{D}_0) = 1$ and $Q(\mathcal{D}_p) = 0$ for $p \geq 0.25$ (no-cloning).

Qubit depolarizing channel

- **Unknown:** $Q(\mathcal{D}_p)$ for $p \in (0, 1/4)$.
- Partial answer for *low noise* ($p \gtrsim 0$):

$$\mathcal{D}_p \approx \mathrm{id} \quad \Longrightarrow \quad Q(\mathcal{D}_p) \approx Q^{(1)}(\mathcal{D}_p) \quad \text{up to } O(p^2 \log p)$$

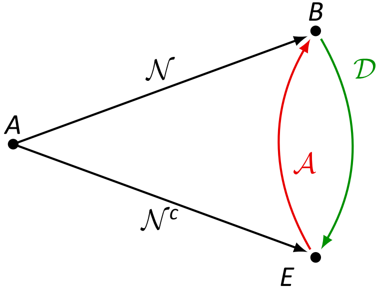
[FL, Leung, Smith 2017] based on [Sutter et al. 2017]

- **Superadditivity:** $Q^{(1)}(\mathcal{D}_p) = 0$ for $p \geq 0.1894$, but $Q^{(1)}(\mathcal{D}_p^{\otimes 3}) > 0$ for $p \lesssim 0.1901$. [DiVincenzo et al. 1998]
- Achieved by repetition code $\sim |0\rangle^{\otimes n} + |1\rangle^{\otimes n}$ (**degenerate** code).
- Result: there are $\mathcal{N}$ and $n \in \mathbb{N}$ s.t. $Q^{(1)}(\mathcal{N}^{\otimes n}) > nQ^{(1)}(\mathcal{N})$.
- For which channels is superadditivity *not possible*, i.e., $Q^{(1)}(\mathcal{N}^{\otimes n}) \leq nQ^{(1)}(\mathcal{N})$?

Table of Contents

Degradable and antidegradable channels

- **Complementary channel** $\mathcal{N}^c : A \to E$ associated to $\mathcal{N}$ models the **leakage of information** to the environment.
- Degradable channels have additive channel coherent information, $Q^{(1)}(\mathcal{N}^{\otimes n}) = nQ^{(1)}(\mathcal{N})$. [Devetak and Shor 2005]
- **Single-letter quantum capacity**: $Q(\mathcal{N}) = Q^{(1)}(\mathcal{N})$.

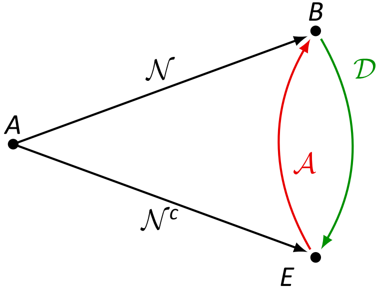


degradable:
$\exists \mathcal{D} : B \to E$ s.t.
$\mathcal{N}^c = \mathcal{D} \circ \mathcal{N}$

antidegradable:
$\exists \mathcal{A} : E \to B$ s.t.
$\mathcal{N} = \mathcal{A} \circ \mathcal{N}^c$

Degradable and antidegradable channels

- Antidegradable channels: $Q(\mathcal{N}) = 0$ due to **no-cloning**.
- Data-processing: $Q^{(1)}(\mathcal{N}) \leq 0$ for antidegradable channels.
- $\mathcal{D}_p$ is antidegradable for $p \geq 1/4$.



degradable:
$\exists \mathcal{D}\colon B \to E$ s.t.
$\mathcal{N}^c = \mathcal{D} \circ \mathcal{N}$

antidegradable:
$\exists \mathcal{A}\colon E \to B$ s.t.
$\mathcal{N} = \mathcal{A} \circ \mathcal{N}^c$

Methods of bounding the quantum capacity

- Given a quantum channel that is *not* degradable/antidegradable, how can we bound its quantum capacity?
- If channel is almost degradable, then capacity $Q(\cdot)$ should be close to $Q^{(1)}(\cdot)$ $\rightarrow$ **approximate degradability** [Sutter et al. 2017]
- Give additional resources (NS/PPT-assistance) to the communicating parties that make quantities more "well-behaved". $\rightarrow$ **SDP bounds**

  [Leung and Matthews 2015; Wang et al. 2017]
- Decompose the channel into degradable/antidegradable parts and use their nice properties.

  $\rightarrow$ [Smith and Smolin 2008] [FL, Datta, Smith 2017]

Decomposition method

- Main insight: $Q(\cdot)$ is **convex** on channels with **additive** $Q^{(1)}(\cdot)$.
  [Wolf and Pérez-García 2007]
- This is true even if the channels in a decomposition are only completely positive, but not necessarily trace-preserving.

Upper bound on $Q(\cdot)$ [FL, Datta, Smith 2018; Yang (in prep.)]

Let $\mathcal{N} = \sum_i p_i \mathcal{E}_i + \sum_i q_i \mathcal{F}_i$, where the $\mathcal{E}_i$ are **degradable** CP maps and the $\mathcal{F}_i$ are **antidegradable**. Then,

$$Q(\mathcal{N}) \leq \sum_i p_i Q^{(1)}(\mathcal{E}_i).$$

- This yields strongest upper bound on $Q(\mathcal{D}_p)$ in high-noise regime (presented at BIID '17).

Optimality of our bound

Main principle

For a channel $\mathcal{N} = (1-\lambda)\mathcal{E} + \lambda\mathcal{F}$, with $\mathcal{E}$ degradable and $\mathcal{F}$ antidegradable, we only count **degradable contributions**:

$$Q(\mathcal{N}) \leq (1-\lambda)Q^{(1)}(\mathcal{E}) = (1-\lambda)Q^{(1)}(\mathcal{E}) + \lambda\,\underbrace{Q^{(1)}(\mathcal{F})}_{=0}$$

- Is there hope to improve our bound by also counting (negative) antidegradable contributions from a joint optimization?

$$(1-\lambda)\max_\varphi I(A\rangle B)_{\mathcal{E}(\varphi)} + \lambda \max_\varphi I(A\rangle B)_{\mathcal{F}(\varphi)} \quad \geq \quad Q(\mathcal{N})$$

$\geq$ ✓ $\qquad\qquad\qquad\qquad\qquad\qquad$ $\geq$?

convexity of $\max(\cdot)$

$$\max_\varphi \left\{(1-\lambda)I(A\rangle B)_{\mathcal{E}(\varphi)} + \lambda\,\underbrace{I(A\rangle B)_{\mathcal{F}(\varphi)}}_{\leq 0}\right\}$$

Optimality of our bound

- Simple case: **flagged channel** ($\mathcal{E}$ deg., $\mathcal{F}$ antideg.)

$$\mathcal{N}_f = (1-\lambda)\mathcal{E} \otimes |0\rangle\langle 0| + \lambda \mathcal{F} \otimes |1\rangle\langle 1| \qquad (*)$$

- Bob can decide which channel occurred by first measuring flag.
- Easy to show:

$$Q^{(1)}(\mathcal{N}_f) = \max_\varphi \left\{ (1-\lambda) I(A\rangle B)_{\mathcal{E}(\varphi)} + \lambda I(A\rangle B)_{\mathcal{F}(\varphi)} \right\}$$

- This is the conjectured upper bound on $Q(\cdot)$!
- Hence, **if true**, any channel $\mathcal{N}_f$ of the form $(*)$ must have **additive coherent information**, since

$$Q^{(1)}(\mathcal{N}_f) \leq Q(\mathcal{N}_f) \leq \text{ conj. upper bound } = Q^{(1)}(\mathcal{N}_f).$$

- **Counterexample!** $\longrightarrow$ Dephrasure channel

Table of Contents

Introducing: the dephrasure channel

Dephrasure channel (**deph**asing + e**rasure**)

For $p, q \in [0,1]$,

$$\mathcal{N}_{p,q}(\rho) := (1-q)((1-p)\rho + pZ\rho Z) + q\,\mathrm{Tr}(\rho)|e\rangle\langle e|.$$

- Dephrasure channel is of the form

$$(1-q)\ \text{deg.} + q\ \text{antideg.}$$

- **Flagged channel**, since $\langle e|\rho|e\rangle = 0$ for all ρ.
- Restrict to $p, q \in [0, 1/2]$ from now on.
- $\mathcal{N}_{p,q}$ is **simple but weird**: exhibits superadditivity of coherent information already for two uses of the channel.

Antidegradability of the dephrasure channel

- **Dephrasure channel:**
  $\mathcal{N}_{p,q}(\rho) = (1-q)((1-p)\rho + pZ\rho Z) + q\,\mathrm{Tr}(\rho)|e\rangle\langle e|$

- **Complementary channel:**

$$\mathcal{N}^c_{p,q}(\rho) = q\,\rho \oplus (1-q) \sum_{x=0,1} \langle x|\rho|x\rangle |\varphi_p^x\rangle\langle\varphi_p^x|,$$

  where $|\varphi_p^x\rangle = \sqrt{1-p}\,|0\rangle + (-1)^x\sqrt{p}\,|1\rangle$.

- Complementary channel is also flagged!

- Want to construct antidegrading map $\mathcal{A}$ such that
  $\mathcal{N}_{p,q} = \mathcal{A} \circ \mathcal{N}^c_{p,q}$.

- **Idea:** unambiguous state discrimination for φ_p^x.

Antidegradability of the dephrasure channel

Unambiguous state discrimination (USD):

- Input: two non-orthogonal states $|\psi_1\rangle$, $|\psi_2\rangle$ with $\langle\psi_1|\psi_2\rangle \neq 0$.
- Design POVM $\{\Pi_1, \Pi_2, \Pi_?\}$ such that
  $\langle\psi_1|\Pi_2|\psi_1\rangle = \langle\psi_2|\Pi_1|\psi_2\rangle = 0$.
- Hence, when receiving outcome ”1” or ”2” we are certain that we have ψ_1 or ψ_2.
- Have to abort if we get outcome ”?”.
- **Optimal measurement:** $\min \Pr(?) = |\langle\psi_1|\psi_2\rangle|$

[Ivanovic 1987; Dieks 1988; Peres 1988]

Antidegradability of the dephrasure channel

Strategy:

$$\mathcal{N}^c_{p,q}(\rho) = q\,\rho \otimes |0\rangle\langle 0|_F + (1-q) \sum_{x=0,1} \langle x|\rho|x\rangle |\varphi_p^x\rangle\langle\varphi_p^x| \otimes |1\rangle\langle 1|_F$$

measure flag F

outcome $|0\rangle\langle 0|$:

post-process ρ

(erase with prob. $1 - \frac{(1-q)(1-2p)}{q}$)

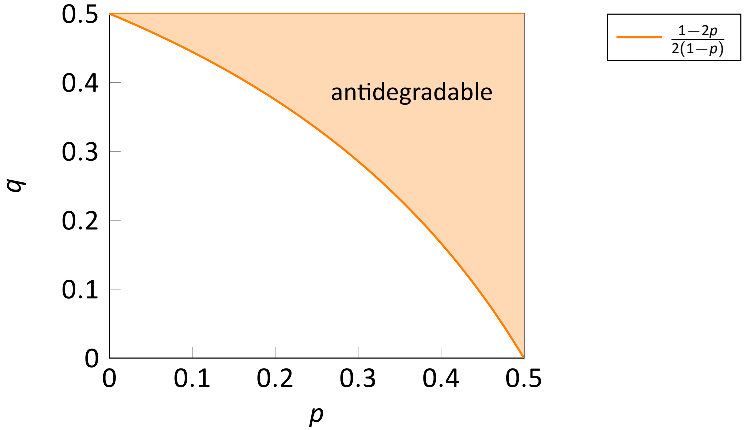
outcome $|1\rangle\langle 1|$:

USD to recover $\langle x|\rho|x\rangle$

(erase on "?")

- Resulting map successfully degrades $\mathcal{N}^c_{p,q}$ to
  $\mathcal{N}_{p,q}(\rho) = (1-q)((1-p)\rho + pZ\rho Z) + q\,\mathrm{Tr}(\rho)|e\rangle\langle e|$.
- Map is completely positive iff $q \geq (1-q)(1-2p)$.

Antidegradability of the dephrasure channel

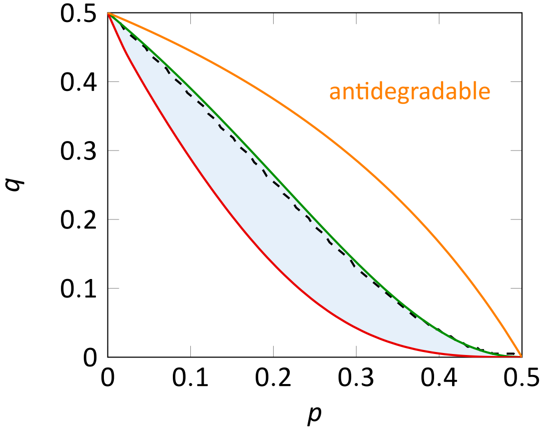


Single-letter coherent information

- For superadditivity of coherent information: need to know $Q^{(1)}(\mathcal{N}_{p,q})$.
- Erasure flag: easy to show that
$$Q^{(1)}(\mathcal{N}_{p,q}) = \max_{|\varphi\rangle_{AA'}} \left\{ (1-q) I(A\rangle B)_{\mathcal{Z}_p(\varphi)} - qS(A)_\varphi \right\},$$
where $\mathcal{Z}_p(\rho)$ is the dephasing channel.
- Form of $Q^{(1)}(\mathcal{N}_{p,q})$ suggests that optimal state is diagonal in Z-basis $\longrightarrow$ true! (simple calculus)
- $$Q^{(1)}(\mathcal{N}_{p,q}) = \max_z \left\{ (1-2q)\, S\begin{pmatrix} (1+z)/2 & 0 \\ 0 & (1-z)/2 \end{pmatrix} - (1-q)\, S\begin{pmatrix} 1-p & z\sqrt{p(1-p)} \\ z\sqrt{p(1-p)} & p \end{pmatrix} \right\}$$

Single-letter coherent information

- $Q^{(1)}(\mathcal{N}_{p,q}) = \max_{|\varphi\rangle_{AA'}} \left\{(1-q)I(A\rangle B)_{\mathcal{Z}_p(\varphi)} - qS(\varphi_A)\right\}.$



$Q^{(1)}(\mathcal{N}_{p,q}) = 0$

inside green line:
optimizing state φ_A
diagonal in Z-basis

red region:
completely mixed state
maximizes $Q^{(1)}(\mathcal{N}_{p,q})$

within blue region:
examples of
superadditivity!

Superadditivity of coherent information

- First thing to try... **weighted repetition code:**

$$|\varphi_n\rangle = \sqrt{\lambda}\,|0\rangle_R|0\rangle_A^{\otimes n} + \sqrt{1-\lambda}\,|1\rangle_R|1\rangle_A^{\otimes n}$$

- For $n = 1$, this is the optimal single-letter code.
- $\mathcal{N}_{p,q}^{\otimes n} = ((1-q)\mathcal{Z}_p + q\,\mathrm{Tr}(\cdot)|e\rangle\langle e|)^{\otimes n}$:

  sum of channels of the form $\mathcal{Z}_p^{\otimes k} \otimes (\mathrm{Tr}(\cdot)|e\rangle\langle e|)^{\otimes n-k}$.
- Coherent information splits up into different erasure patterns, since $S(\sum_i p_i\rho_i \otimes |i\rangle\langle i|) = \sum_i p_i S(\rho_i) + H(\{p_i\})$.
- Repetition code: all partial erasures cancel.
- Easy: compute action of dephasing $\mathcal{Z}_p^{\otimes n}$ on repetition code.

Superadditivity of coherent information

- (Almost) closed formula for repetition code ($\varphi_n = \varphi_n(\lambda)$):

$$Q^{(1)}(\varphi_n, \mathcal{N}_{p,q}^{\otimes n}) = ((1-q)^n - q^n)\, h(\lambda) \\ -(1-q)^n \left(1 - u \operatorname{artanh} u - \tfrac{1}{2} \log\left(1 - u^2\right)\right).$$

- $u = u(\lambda, p, n) = \sqrt{1 - 4\lambda(1-\lambda)(1-(1-2p)^{2n})}$.

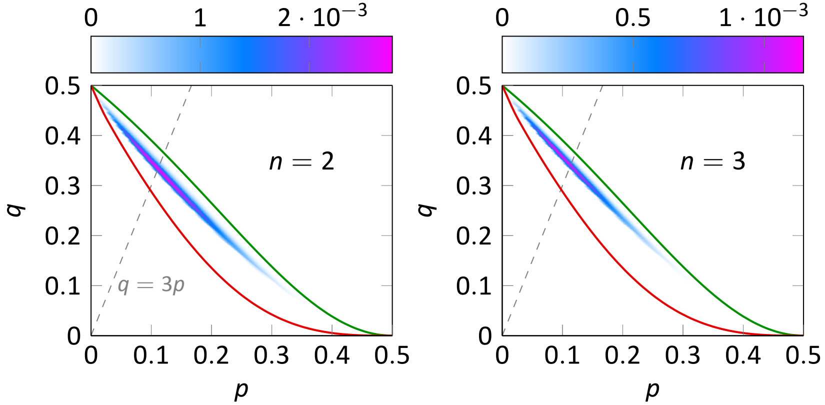
- $h(\lambda) = -\lambda \log \lambda - (1-\lambda)\log(1-\lambda)$ is the binary entropy of λ.

- To get superadditivity, maximize over parameter λ (weight of logical $|0\rangle$ in the repetition code).

Superadditivity of coherent information

- Weighted repetition code $|\varphi_n\rangle := \sqrt{\lambda}|0\rangle^{\otimes n+1} + \sqrt{1-\lambda}|1\rangle^{\otimes n+1}$.
- Plot for $n = 2, 3$ of the non-negative part of

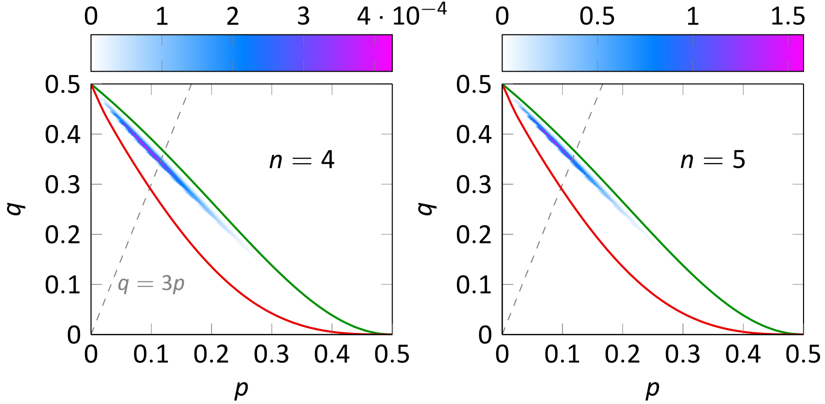
$$\frac{1}{n}\max_{\lambda} I(A\rangle B^n)_{\mathcal{N}_{p,q}^{\otimes n}(\varphi_n)} - Q^{(1)}(\mathcal{N}_{p,q})$$



Superadditivity of coherent information

- Weighted repetition code $|\varphi_n\rangle := \sqrt{\lambda}|0\rangle^{\otimes n+1} + \sqrt{1-\lambda}|1\rangle^{\otimes n+1}$.
- Plot for $n = 4, 5$ of the non-negative part of

$$\frac{1}{n} \max_{\lambda} I(A\rangle B^n)_{\mathcal{N}_{p,q}^{\otimes n}(\varphi_n)} - Q^{(1)}(\mathcal{N}_{p,q})$$



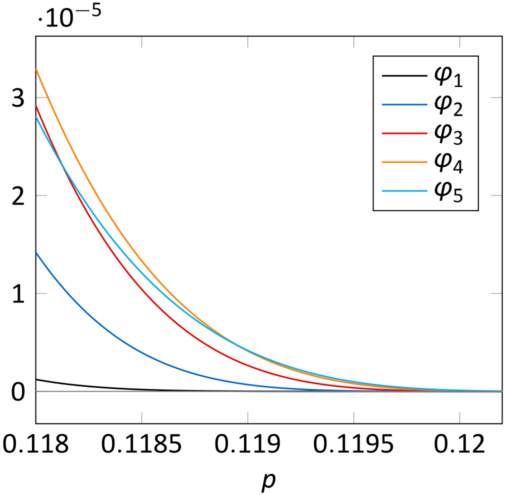
Superadditivity of coherent information

- Superadditivity also holds in the ”extreme form”:
  $$Q^{(1)}(\mathcal{N}_{p,q}) = 0 \quad \text{but} \quad Q^{(1)}(\varphi_n, \mathcal{N}_{p,q}^{\otimes n}) > 0$$
  for $n \geq 2$ and some $p, q \longrightarrow$ **increased threshold**.
- We also have more elaborate codes achieving superadditivity, for example for $n = 3$:
  $$|\chi_3\rangle := |00\rangle|00\rangle \otimes |\psi_1\rangle + |11\rangle|11\rangle \otimes |\psi_1\rangle + |01\rangle|01\rangle \otimes |\psi_2\rangle + |10\rangle|10\rangle \otimes X|\psi_2\rangle,$$
  for some pure states $|\psi_i\rangle$.
- We also found good codes using a **neural network state ansatz**.

  [Bausch, FL; arXiv:1806.08781]
- Not clear whether optimal codes for $n \geq 2$ are diagonal in Z-basis (true for $n = 1$)!

Superadditivity of coherent information

- Plot for $\mathcal{N}_{p,3p}$ along diagonal $(p, 3p)$
- Threshold is increased by repetition codes φ_n.



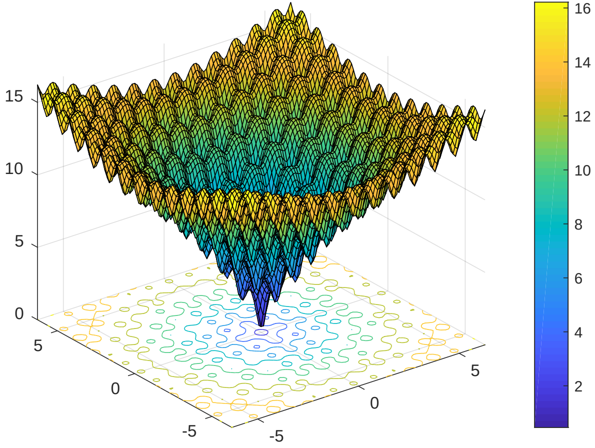
Numerical optimization techniques

- Easy observation: $I(A\rangle B)_{\psi\otimes\mathcal{N}(\varphi)} = 0$ for a product input state $|\psi\rangle_A \otimes |\varphi\rangle_{A'}$.
- Hence, **many local maxima** in high-noise regime where most states have negative channel coherent information.
- Gradient is likely to get stuck $\longrightarrow$ gradient-free optimization?
- Many biology-inspired examples: genetic algorithms, artificial bee colonization, **particle swarm optimization** (PSO)
- Idea of PSO:
  - Send out N particles, each probing the landscape.
  - Each particle records personal best function value, and all know the global swarm best.
  - In each iteration, particle velocity is updated with weights towards personal best, global best, and inertial movement.

Particle swarm optimization

Ackley function: $f(x, y) =$

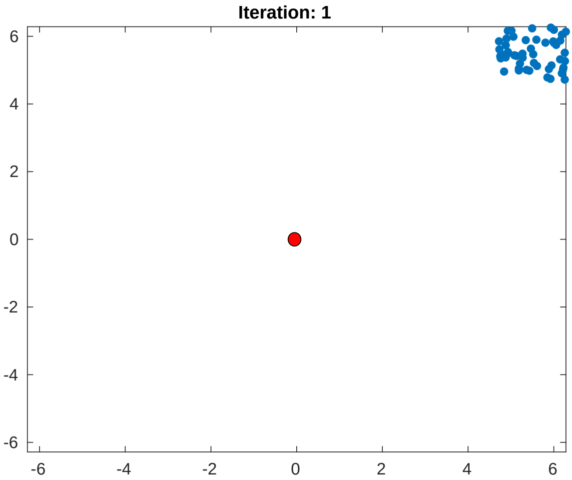
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

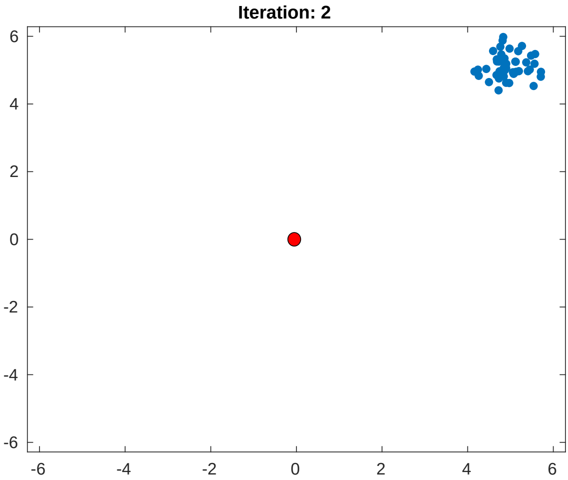
Ackley function: $f(x, y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



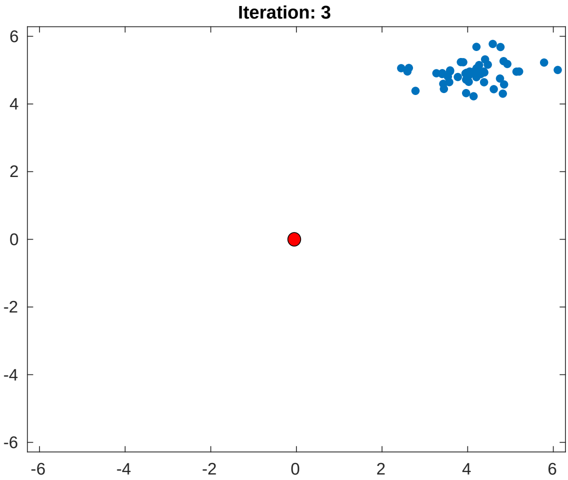
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



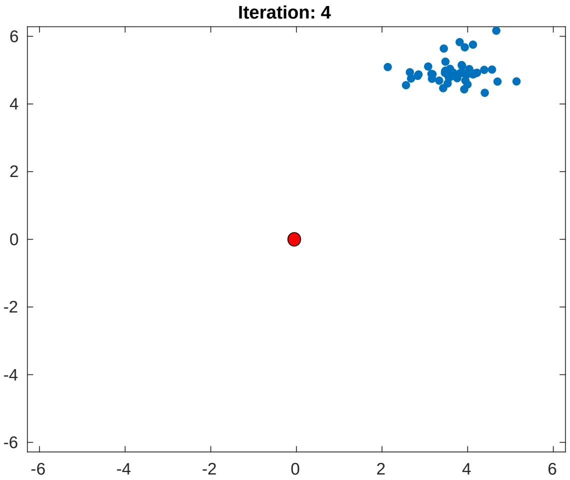
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right]-\exp\left[0.5(\cos 2\pi x+\cos 2\pi y)\right]+e+20$$



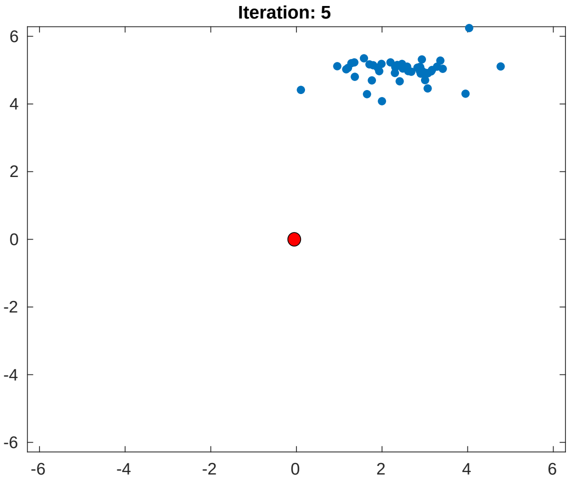
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right]-\exp\left[0.5(\cos 2\pi x+\cos 2\pi y)\right]+e+20$$



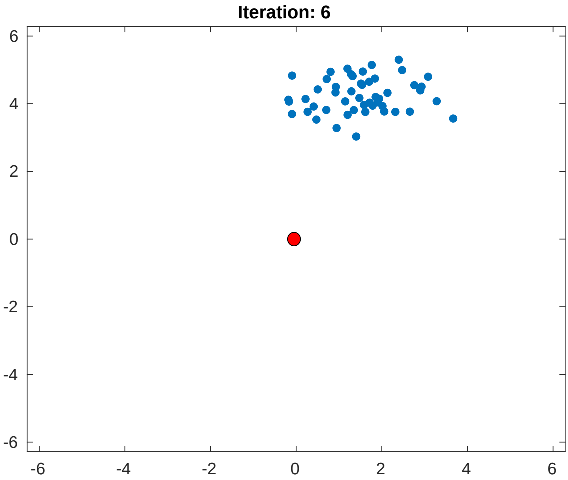
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



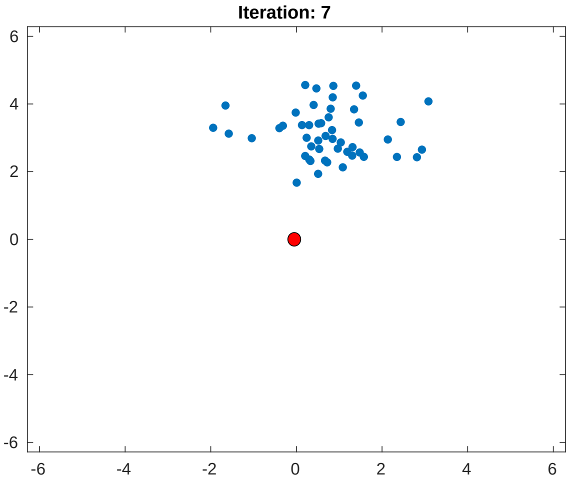
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



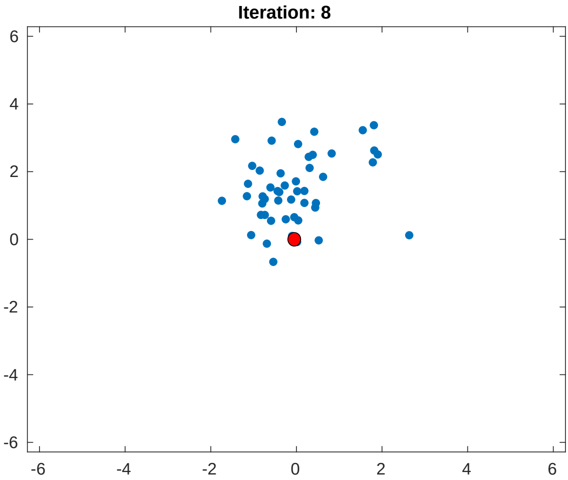
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



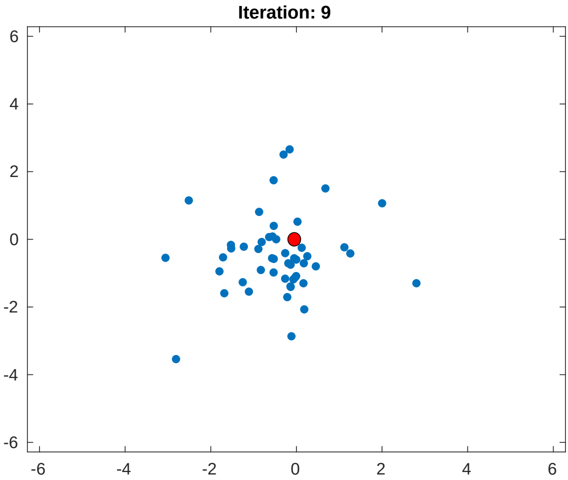
Particle swarm optimization

Ackley function: $f(x,y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



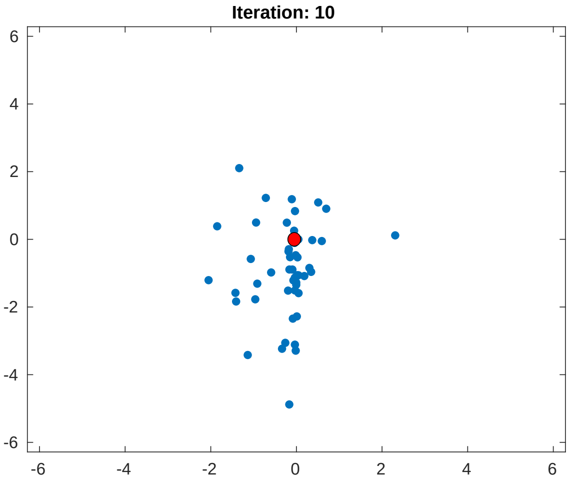
Particle swarm optimization

Ackley function: $f(x,y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



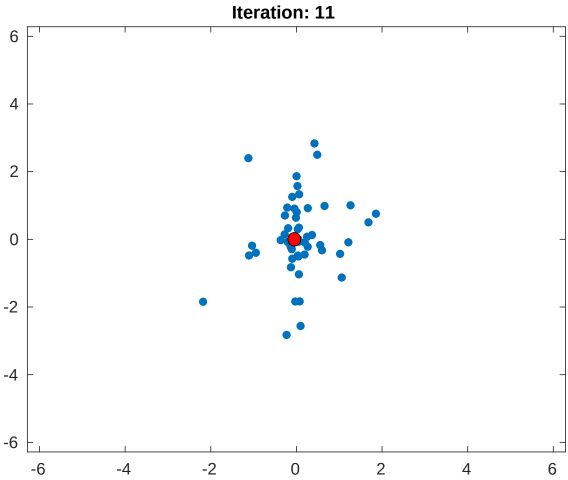
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2 + y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

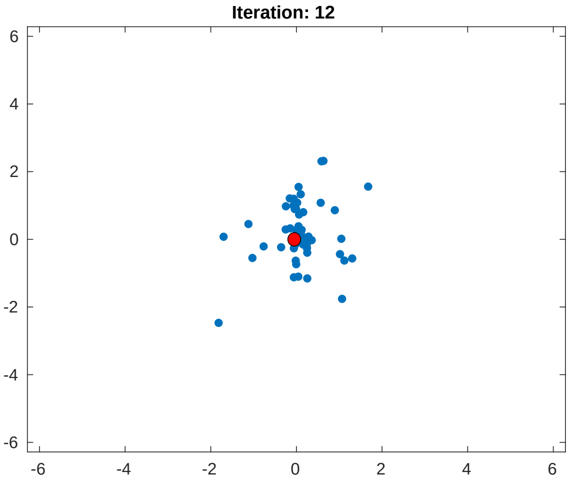
Ackley function: $f(x,y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

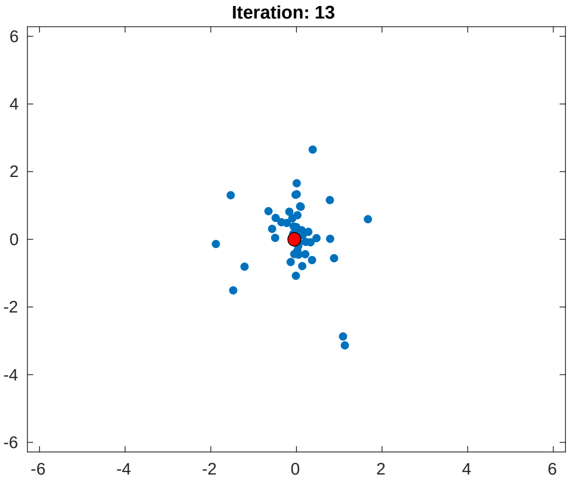
Ackley function: $f(x,y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



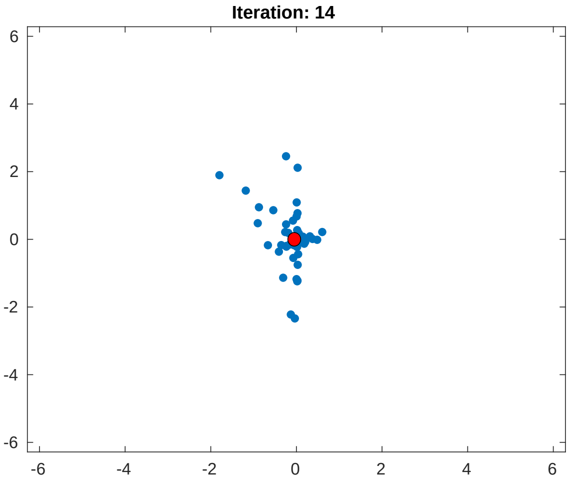
Particle swarm optimization

Ackley function: $f(x,y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

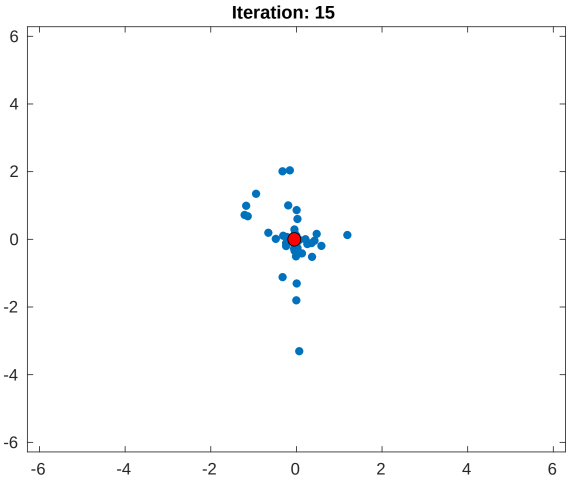
Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

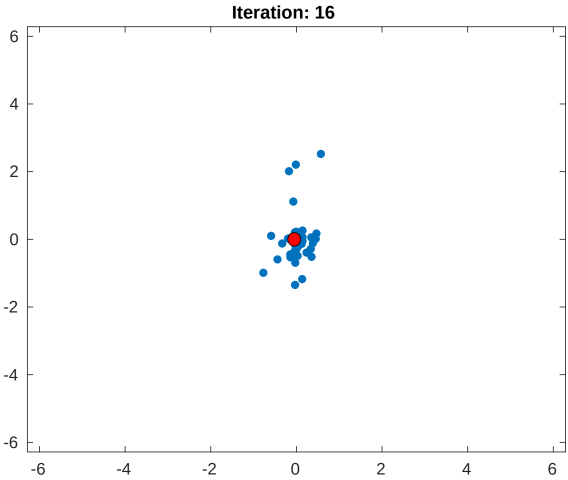
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

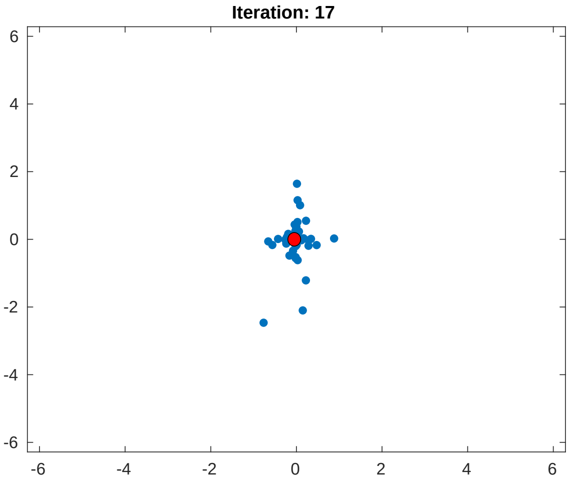
$$-20 \exp\left[-0.2\sqrt{0.5(x^2 + y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

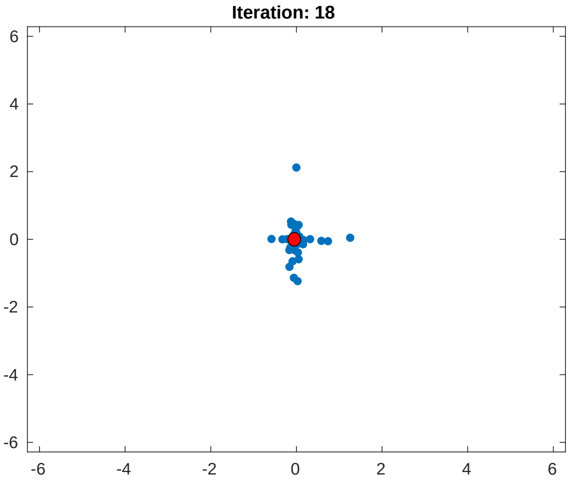
Ackley function: $f(x, y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

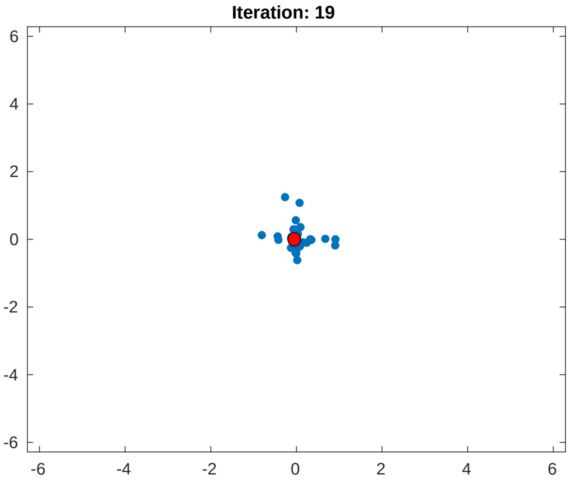
Ackley function: $f(x, y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

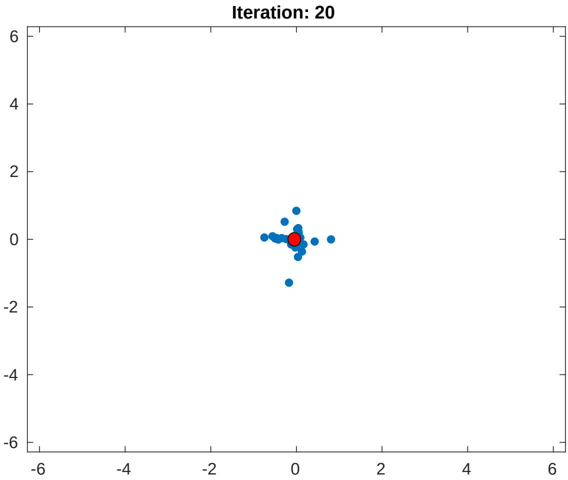
$$-20 \exp\left[-0.2\sqrt{0.5(x^2 + y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

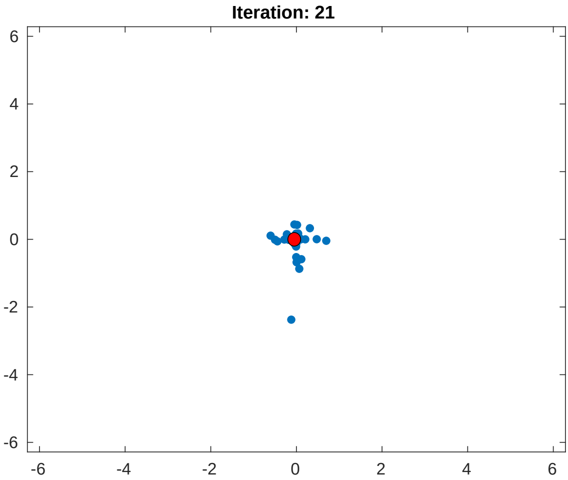
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

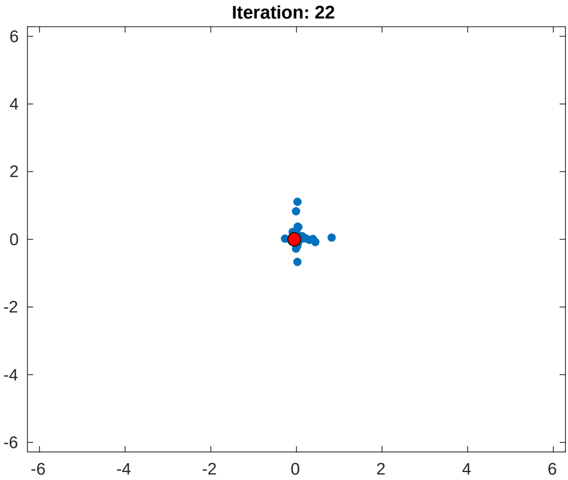
Ackley function: $f(x,y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right]-\exp\left[0.5(\cos 2\pi x+\cos 2\pi y)\right]+e+20$$



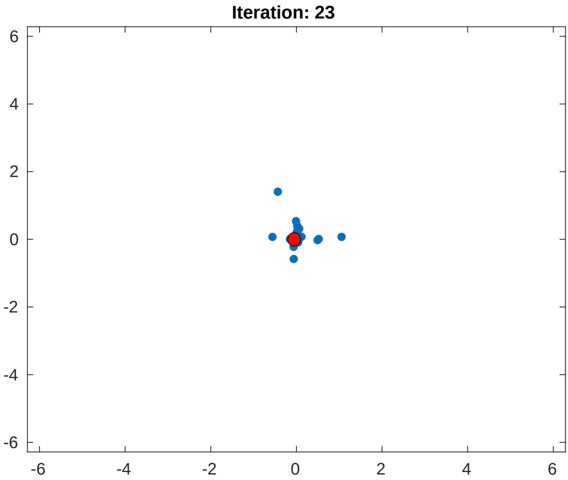
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



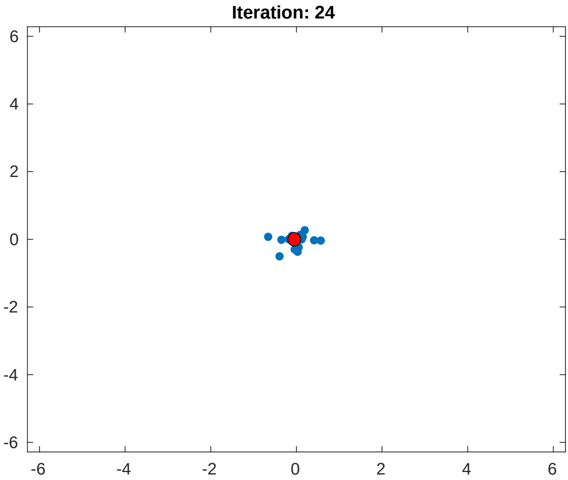
Particle swarm optimization

Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2 + y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

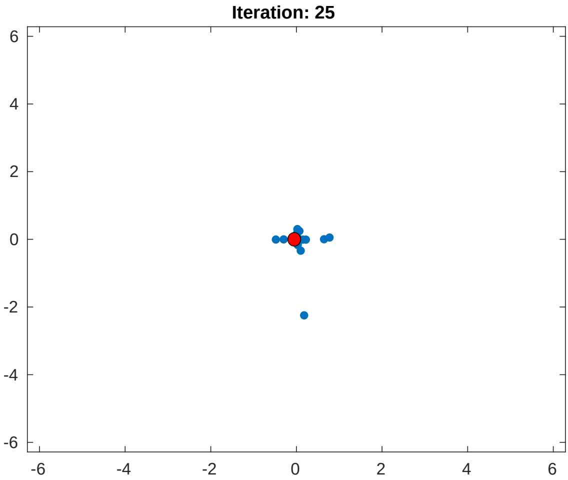
Ackley function: $f(x, y) =$
$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

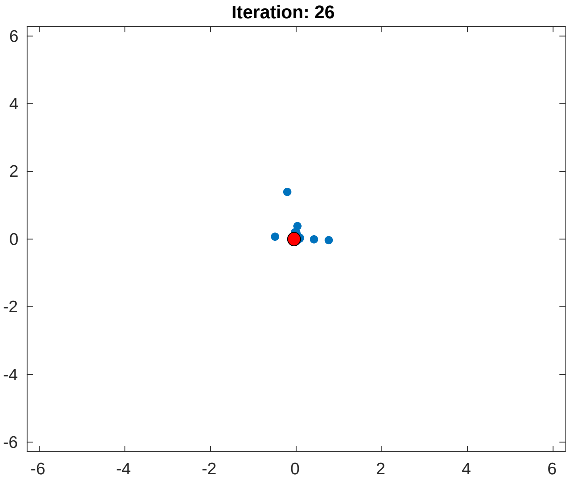
Ackley function: $f(x, y) =$

$$-20 \exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

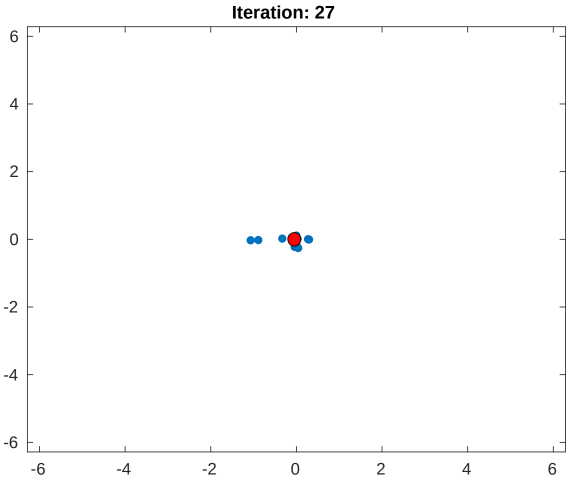
Ackley function: $f(x, y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

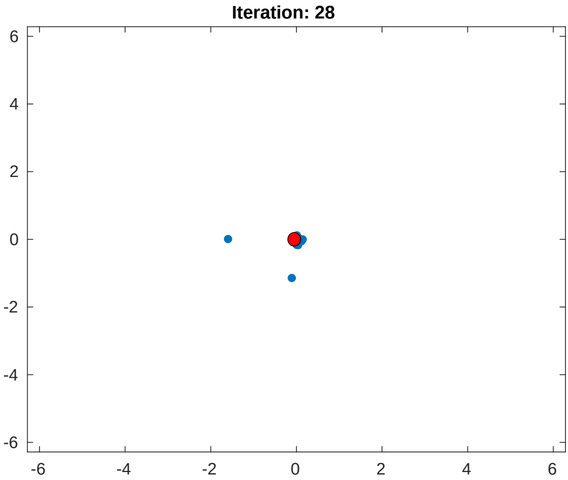
Ackley function: $f(x, y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right]-\exp\left[0.5(\cos 2\pi x+\cos 2\pi y)\right]+e+20$$



Particle swarm optimization

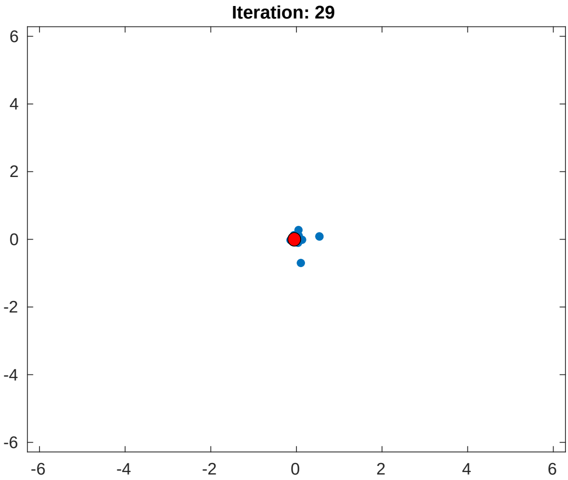
Ackley function: $f(x,y) =$
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

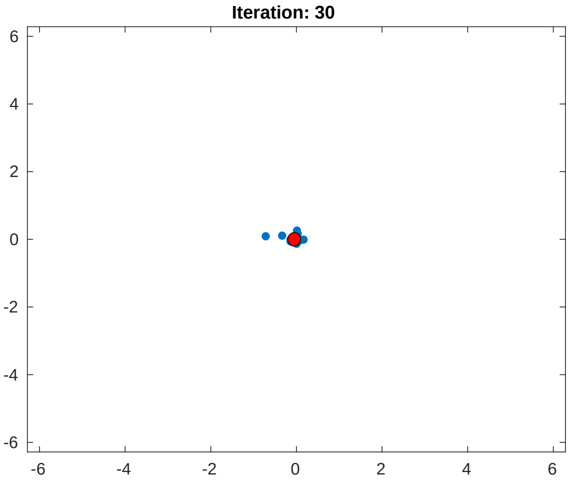
$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Particle swarm optimization

Ackley function: $f(x, y) =$

$$-20\exp\left[-0.2\sqrt{0.5(x^2+y^2)}\right] - \exp\left[0.5(\cos 2\pi x + \cos 2\pi y)\right] + e + 20$$



Private capacity of a quantum channel

- **Private capacity** $P(\mathcal{N})$: highest rate of **private classical communication** between Alice and Bob

- Coding theorem: [Devetak 2005; Cai et al. 2004]

$$P(\mathcal{N}) = \lim_{n\to\infty} \frac{1}{n} P^{(1)}(\mathcal{N}^{\otimes n})$$

with the **private information**

$$P^{(1)}(\mathcal{N}) = \max_{\{p_i,\rho_i\}} [I(X;B)_\rho - I(X;E)_\rho].$$

- Private information can be superadditive, [Smith et al. 2008]

$$P^{(1)}(\mathcal{N}^{\otimes n}) > nP^{(1)}(\mathcal{N}).$$

Private capacity of a quantum channel

- Quantum information transmission is necessarily private:

$$P(\mathcal{N}) \geq Q(\mathcal{N}).$$

- Also holds for information quantities: $P^{(1)}(\mathcal{N}) \geq Q^{(1)}(\mathcal{N})$.
- For degradable channels: [Smith 2008]

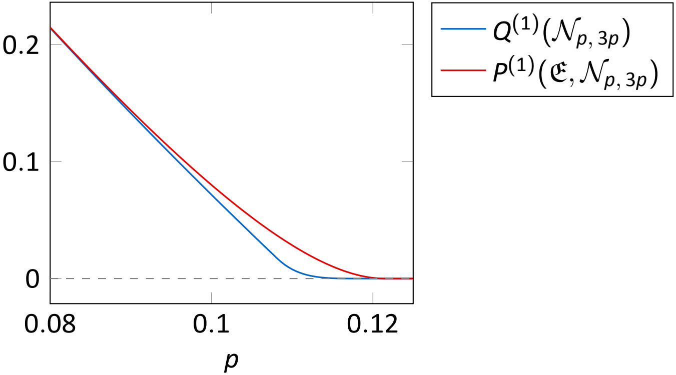
$$P(\mathcal{N}) = Q(\mathcal{N}) = Q^{(1)}(\mathcal{N}) = P^{(1)}(\mathcal{N}).$$

- For antidegradable channels: $P(\mathcal{N}) = 0 = Q(\mathcal{N})$.
- There are channels with $Q(\mathcal{N}) = 0$ and $P(\mathcal{N}) > 0$ (e.g. entanglement-binding channels).
- Leads to superactivation of $Q(\cdot)$: $\exists \mathcal{N}_1, \mathcal{N}_2$ with $Q(\mathcal{N}_i) = 0$ but $Q(\mathcal{N}_1 \otimes \mathcal{N}_2) > 0$. [Smith and Yard 2008]

Separation of private and coherent information

- Numerical investigations suggest the following is an **optimal private ensemble**: $(|\pm\rangle \sim |0\rangle \pm |1\rangle)$

$$p_1 = \tfrac{1}{2}, \quad \rho_1 = \lambda|+\rangle\langle+| + (1-\lambda)|-\rangle\langle-|$$
$$p_2 = \tfrac{1}{2}, \quad \rho_2 = \lambda|-\rangle\langle-| + (1-\lambda)|+\rangle\langle+|$$



Separation of private and coherent information

- **Separation of capacities?** $P(\mathcal{N}_{p,q}) > Q(\mathcal{N}_{p,q})$ $(= 0)$?
- **superadditivity of private information?**
  $P^{(1)}(\mathcal{N}_{p,q}^{\otimes n}) > nP^{(1)}(\mathcal{N}_{p,q})$?

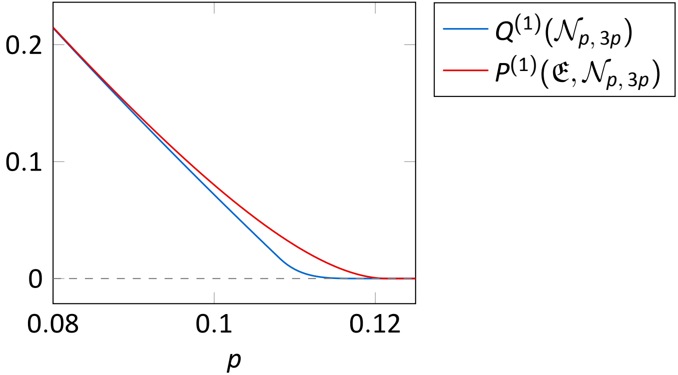


Table of Contents

Summary

- Discussed an upper bound on quantum capacity of a channel based on **decomposition into degradable and antidegradable channels**.

- Question of optimality leads to the **dephrasure channel**

$$\mathcal{N}_{p,q}(\rho) = (1-q)((1-p)\rho + pZ\rho Z) + q\,\mathrm{Tr}(\rho)|e\rangle\langle e|.$$

- Dephrasure channel checks a lot of marks on ”weirdness chart”:
  - superadditivity of coherent information for two uses
  - separation of private and coherent information
  - possible superadditivity of private information?

Outlook

- Some open questions:
  - Formula for single-letter private information (needed to show superadditivity)?
  - Tight upper bounds on quantum capacity?
  - Increase threshold up to region of antidegradability?
- Further effects of superadditivity? $\longrightarrow$ superadditivity in classical communication with limited entanglement assistance (ongoing work with Elton Zhu, Quntao Zhuang)
- Can we understand the superadditivity of coherent information in the light of the recent work on α-bit capacities?

References

Bausch, J. and F. Leditzky (2018). *arXiv preprint*. arXiv: 1806.08781 [quant-ph].

Cai, N. et al. (2004). *Problems of Information Transmission* 40.4, pp. 318–336.

Devetak, I. (2005). *IEEE TIT* 51.1, pp. 44–55. arXiv: quant-ph/0304127.

Devetak, I. and P. W. Shor (2005). *CMP* 256.2, pp. 287–303. arXiv: quant-ph/0311131 [quant-ph].

DiVincenzo, D. P. et al. (1998). *Physical Review A* 57.2, p. 830. arXiv: quant-ph/9706061.

Dieks, D. (1988). *Physics Letters A* 126.5, pp. 303–306.

Ivanovic, I. (1987). *Physics Letters A* 123.6, pp. 257–259.

Leditzky, F. et al. (2018a). *IEEE TIT* 64.7, pp. 4689–4708. arXiv: 1701.03081 [quant-ph].

Leditzky, F. et al. (2018b). *Physical Review Letters* 120.16, p. 160503. arXiv: 1705.04335 [quant-ph].

Leung, D. and W. Matthews (2015). *IEEE TIT* 61.8, pp. 4486–4499. arXiv: 1406.7142 [quant-ph].

Lloyd, S. (1997). *Physical Review A* 55.3, p. 1613. arXiv: quant-ph/9604015.

Peres, A. (1988). *Physics Letters A* 128.1, p. 19.

Shor, P. W. (2002). *MSRI Workshop on Quantum Computation*. Berkeley, CA, USA.

Smith, G. (2008). *Physical Review A* 78.2, p. 022306. arXiv: 0705.3838 [quant-ph].

Smith, G. and J. A. Smolin (2008). *2008 IEEE ITW*. IEEE, pp. 368–372. arXiv: 0712.2471 [quant-ph].

Smith, G. and J. Yard (2008). *Science* 321.5897, pp. 1812–1815. arXiv: 0807.4935 [quant-ph].

Smith, G. et al. (2008). *Physical Review Letters* 100.17, p. 170502. arXiv: quant-ph/0607018.

Sutter, D. et al. (2017). *IEEE TIT* 63.12, pp. 7832–7844. arXiv: 1412.0980 [quant-ph].

Wang, X. et al. (2017). *arXiv preprint*. arXiv: 1709.00200 [quant-ph].

Wolf, M. M. and D. Pérez-García (2007). *Physical Review A* 75.1, p. 012303. arXiv: quant-ph/0607070.

Yang, D. Manuscript in preparation.

Thank you very much for your attention!

Announcement: QIP 2019

QIP 2019: Jan 14-18, 2019 at University of Colorado Boulder

Website: `jila.colorado.edu/qip2019`

Local organizers: Felix Leditzky, Graeme Smith

Program committee chair: Matthias Christandl

Submission deadline: sometime in September (TBD)